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di Gregory Landini

In an amusing opening to his influential paper "On What there Is," Quine writes: "A curious thing about the ontological problem is its simplicity. It can be put in three Anglo-Saxon monosyllables: 'What is there?' It can be answered, moreover, in a word -'Everything' - and everyone will accept this answer as true. However, this is merely to say that there is what there is. There remains room for disagreement over cases; and so the issue has stayed alive down the centuries."[1] The curious thing about this opening, is that it is so very far from Russell's views on the nature of ontology.

Russell's ontological development has been the subject of rebuke from Broad who, perhaps in jest, accused him of being a flighty philosopher for publishing a new system of philosophy every few years. Quine's penchant for desert landscapes, his feeling that intensional entities are "creatures of darkness," might be placed in contrast to the many denizens of Russell's different ontological theories over the years - non-existents such as Pegasus, classes, propositions, denoting concepts, negative facts, general facts. Yet it is Russell, not Quine, who is responsible for making logic the central component in questions of ontology. The importance of the new quantificational logic developed by Frege and Russell shows up surreptitiously when Quine cagily answers "Everything" to the question "What is there? " Russell set forth the connection between ontology and quantification long before Quine coined the maxim: *To be is to be the value of a variable.*[2]

Russell, however, would not have accepted the question, "What is there?" This seems just a variant of "What exists?" and for Russell (post 1905) this has no truth-conditions. It is a pseudo-question.

That some ontological questions are pseudo-questions is perhaps one of the most important of Russell's discoveries. Before assessing whether a given natural language argument is valid, we must be sure we have property represented the syntactic structures which capture the truth-conditions of the premises and conclusion composing it. The endeavor to uncover the truth-conditions for a given statement is a quest to uncover what Russell calls "logical form." This is not always an *a priori* matter. Every argument, after all, will be deemed invalid if we represent its syntactic form as

P, Q, R…. .; Therefore C

[1] W.V.O. Quine, "On What There Is," in *From a Logical Point of View* (Harvard University Press, 1964), pp.1-19.
[2] W.V.O. Quine, "On What There Is," p. 15.

To properly portray the truth-conditions, we may well have to know something about the empirical sciences and mathematics involved. Consider the argument:

Timber wolves are disappearing
Petrushka is a timber wolf.
Therefore, Petrushka is disappearing.

What is being said by the first premise? If we naively take the premise to say: Everything is such that if it is a timber wolf then it is disappearing then we could apply it to Petrushka and yield. If Petrushka is a timber wolf then Petrushka is disappearing. But the premise does not have this truth-condition. The truth-condition for "φ's are disappearing" is this: The number of "φ's born during a current temporal interval in a region is less than the number of "φ's that died during that current temporal interval in that region.

In the sense in which it is true that timber wolves are *disappearing*, the sentences "Petruska is disappearing" is ungrammatical. In Russell's view, the statement "Men are numerous" has a logical form similar to the statement "Timber wolves are disappearing." Consider the argument

Men are numerous.
Socrates is a man.
Therefore, Socrates is numerous.[3]

The odd conclusion could only follow from the premises if we adopt the perverse reading that "Men are numerous" is true just when everything is such that if it is a man then it is numerous. The proper truth-condition is that the number of men is much greater than zero. Similarly, consider:

Men are existing.
Socrates is a man.
Therefore, Socrates is existing.[4]

Once again it would be perverse to think that the truth condition of "Men are existing", is that everything is such that if it is a man then it is existing. The truth condition is that the number of men is not zero. Assertions of existence in ordinary language are numeric and thus their truth-conditions are given quantificationally.

Of course, in ordinary communication we use "existence" as if it were a property. We say that God exists and deny Pegasus exists. But in his 1905 paper "On Denoting," Russell showed the way to give quantificational truth conditions for such ordinary language sentences involving ordinary proper names and ordinary definite descriptions. On Russell's view, to give the truth-condition for a statement such as "Pegasus exists" we replace the ordinary name "Pegasus" with an ordinary definite description "the A" and then assert $(\exists x)(\mathrm{A}y \equiv_{\mathrm{y}} y = x)$.

This says that the number of objects that are A is exactly one. (Otherwise put, it says that one and only one object is an A.) In Russell's view, it is a perverse logic which offers truth-conditions for "Pegasus exists" as "E(Pegasus), where "E$x$" means "$x$ exists." Russell does

[3] Bertrand Russell, "The Philosophy of Logical Atomism," in Robert Marsh ed., *Bertrand Russell: Logic and Knowledge Essays 1901-1950* (London: Allen and Unwin, 1977), p. 233.
[4] Op. Cit., p.233.

allow us to write $E!(\iota x A x)$ as a convenient short hand, so that we don't have to write the complicated expression $(\exists x)(Ay \equiv_y y = x)$. But this convenience respects his view that the truth-conditions for existence statements are always numeric and quantificational. Russell's account of the nature of statements of existence is an insight that dispenses with a great tendency for speculative philosophical ontology— some of cosmological proportion. Russell made this point in his radio debate with Father Copleston[5] :

> **Copleston:** But are you going to say that we can't, or we shouldn't even raise the question of the existence of the whole of this sorry scheme of things—of the whole universe?
>
> **Russell:** Yes. I don't think there's any meaning in it at all. I think the word "universe" is a handy word in some connections, but I don't think it stands for anything with meaning.

The sentence "Why does the universe exist?" is just a figurative way to phrase the sentence "Why does something exist?" But in Russell's view, this has no truth-conditions. This is not to say, however, that there are no truth-conditions for "Why does something with mass exist?" One can, with trepidation about the legitimacy of the physical notion of *mass,* ask why something has mass. But the question "Why is there something rather than nothing?" has no-truth conditions. A philosophical conundrum captivating some of the greatest of minds for centuries is but a pseudo-question.

Russell's discovery has a very marked impact on questions of philosophical ontology. Indeed, Ramsey spoke of it as a "paradigm" for a new scientific philosophy.[6] In Russell's view, logically proper scientific languages (and theories) supplant ontological theories that confusedly infuse science with doctrines of speculative metaphysics. Russell's philosophy endeavors to solve problems in a given theory by abandoning the ontology of the old theory and offering a logical reconstruction of its laws within a new *ontological* framework. Putting the point starkly, Russell writes: "…very philosophical problem, when it is subjected to the necessary analysis and purification, is found either to be not really philosophical at all, or else to be, in the sense in which we are using the word, logical."[7] Logic is the essence of Russell's philosophy and ontology.

## *1. Russell's Scientific Method in Philosophy*

What was Russell's new scientific method in philosophy? In his book, *Our Knowledge of the External World as a Field for Scientific Method in Philosophy* (1914) Russell endeavored to apply his new conception of philosophy to the problem of matter. Later the new philosophy would be called "logical atomism." Russell characterizes his program as one governed by a "supreme maxim of all scientific philosophizing," namely this:

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[55] See Bertrand Russell, "A Debate on the Existence of God," in ed., Al Seckel, *Bertrand Russell on God and Religion* (Buffalo: Prometheus Books, 1986), pp. 123-146.

[6] Frank Plumpton Ramsey, " Philosophy" in ed., R. B. Braithwaite, *The Foundations of Mathematics and other logical essays* (New York: Harcourt, Brace and Co., 1931), p. 263.fn.

[7] Bertrand Russell, *Our Knowledge of the External World* (London: Allen & Unwin, 1969), p. 42.

"Wherever possible, logical constructions are to be substituted for inferred entities."[8]

Unfortunately, in his article "Logical Atomism," Russell stated the maxim in an epistemic form:

> "Wherever possible, substitute constructions out of known entities for inferences to unknown entities."

This form of the maxim has mislead interpretations ever since. Interpretations present Russell's new philosophy as advocating a form of empiricist reduction. Pears succinctly represents this view. in the following:

> 'Logical atomism' is Russell's name for the theory that there is a limit to the analysis of factual language, a limit at which all sentences will consist of words designating simple things… His theory of knowledge led him to claim that the only simple particulars that we know are sense-data, and that the only simple qualities and relations we know are certain qualities and relations of sense-data. Their simple qualities and relations are those with which we have to achieve acquaintance in order to understand the words designating them. This fixes the character of his logical atomism. It is a version of empiricism and it uses a criterion of simplicity based on the exigencies of learning meanings. …The doctrine of forced acquaintance is the foundation of Russell's logical atomism.[9]

Russell allegedly operates with a criterion of simplicity that allows him to identify logical atoms as things with which we are familiar, namely sense-data and their properties. [10] The foundation of Russell's atomism, on this view, is an empiricist principle of acquaintance. It takes sense-data and universals as its logical atoms and offers an empiricistic construction of factual language. Even the logical constructions of *Principia* have been interpreted as a form of reduction in the spirit of empiricism. For instance, Ayer writes:

> …when Russell spoke of an object as a logical fiction, he did not mean to imply that it was imaginary or nonexistent… Similarly, in the period during which Russell held that physical objects were logical constructions, he did not wish to suggest that they were unreal in the way that gorgons are unreal.. Logical fictions do indeed exist, but only in virtue of the existence of the elements out of which they are constructed. As Russell put it, they are not part of the ultimate furniture of the world." [11]

Ayer assumes that *Principia* is a theory of ramified types of entities ("propositional functions"), and that classes, numbers, and the like are logical fictions in the sense that they are reductively identified with an ontology of attributes in intension (i.e., "propositional functions"). Russell's subsequent application of the techniques to matter and mind are understood in the same way. Rocks and mountains (material objects) do exist according to reductive empiricism— their being orderings of sense-experiences notwithstanding.

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[8] Bertrand Russell, "On the Relation of Sense-Data to Physics," *Scientia* vol. 4, 1914. Reprinted in *Mysticism and Logic and Other Essays,* (Barnes & Noble, 1976), p. 115.
[9] David Pears, *The False Prison, Vol. 1*, p. 63.
[10] *Ibid.*, p. 68
[11] A. J. Ayer, "Bertrand Russell as a Philosopher" in *The Meaning of Life* (New York: MacMillan, 1990), p. 152.

Russell's insistence that he is offering a "no-classes" theory, that there is no matter, etc., challenges this interpretation. The lesson Russell drew from his work in logic is that the calculus for logic must proceed without the ontological assumption that every open formula comprehends a class. By December of 1905 Russell had used his new theory of definite descriptions to form a wonderfully creative reconstruction of the first principles of logic. The reconstruction enables one to emulate a type-theory of classes without its ontology. It is not reductive identity that Russell had in mind when he said that classes are "logical fictions." Just as Maxwell's equations for the propagation of electromagnetic waves are retained in Einstein's no-aether theory of relativity, Russell's eliminativism is a reconstruction that preserves the laws of a type-theory of classes in a no-classes theory. For Russell, classes do go the way of the Gorgon sisters.

Russell's examples of his scientific method in philosophy do not corroborate the interpretation that it is an ontological reduction to knowable entities based on a principle of acquaintance. Russell's examples are not cases of an empiricist analysis of factual language—as if Einstein's theory of space-time is an analysis of the 19th -century factual language of "simultaneity"or Weierstrass's reconstruction of limits is an analysis of the use of the word "limit" as used in the practice of applying the calculus. How then shall we understand Russell's scientific method in philosophy?

Perhaps it is best to begin with a brief point about the philosophy of science. It is here that we find the entanglement of conceptions of empiricism, semantics and ontology. The mechanical philosophy[12] of seventeenth century science rejected the Aristotelian teleological explanations together with its ontology of natural states and places, metaphysical essences, entelechies, drives, humours, vital forces, and the like. An Aristotelian would say that a planet's natural place is in the cosmos and it natural state is to move at a uniform rate in a circle. In contrast, natural terrestrial motions are non-uniform in rate and rectilinear. The natural state being rest, motion in the terrestrial sphere requires explanation since some force, conatus or animus must initiate it and sustain it throughout. Aristotelian's thought that a rock's rate of fall after being dropped is an expression of an entelechy (disposition) within it to find its natural place and return to its natural state of rest. The attraction of iron to the lode stone is a manifestation of its internal disposition (attraction). The Copernican revolution with its moving earth destroyed Aristotelian entelechies, natural states and places. The terrestrial and celestial realms were unified under new laws of motion, and a new conception of science as mechanical was born. The principle of demarcation of the new science was the requirement that only mechanical processes, governed by quantitative mathematical laws, are properly scientific. For example, it is a scientific hypothesis according to mechanists that the lode stone spews forth screw shaped particles which permeate the iron, pulling the two together. The conjecture that it contains within it a dispositional force of magnetic *attraction* is not scientific. The new mechanical science is antithetical to Aristotelian teleological explanations in terms of 'forces,' and drives.

Strict adherence to the mechanical model, however, impeded progress in physics and seemed intractable in chemistry and biology. Eschewing Harvey's vital force which drives systole, Descartes could embrace Harvey's thesis of the circulation of the blood only by imagining the heart's pumping to be driven by pressures akin to those found in a tea kettle. The blood heats in the heart and (as if like steam) the increase in pressure pushes the blood through the arteries; its cooling produces condensation and returns it to the heart through the veins. The mechanist rejection of 'forces' as occult vestiges of Aristotelianism threatened Ga-

[12] Not all mechanists were atomists. Descartes accepted infinite divisibility and thus imagined a fluid dynamics for his *res extensae*.

lileo's work in dynamics. Newton ingeniously offered a synthesis.[13] Forces are legitimate in science only if precise mathematical laws govern them, laws that admit of empirical test. We can write quantitative equations for the relationships and test them. Occult as it is by the lights of mechanists, the force of gravitational attraction at a distance given by Newton's famous formula

$$F = G\left(\frac{m_1 \times m_2}{d^2}\right)$$

can be measured and tested.[14] Newton's famous dictum, *Non Fingo Hypotheses* (I do not make hypotheses) aptly distinguished his natural philosophy from the mechanical approach that preceded it.[15] In Newton view, the criterion of a hypothesis's being scientific is its amenability to empirical test.

Empiricist reductions in philosophy became popular with the success of Newton's physics at excluding Aristotelian metaphysics. The wide success of Newton's conception of science encouraged philosophers to take his methods as the paradigm for rationality itself.

Philosophers such as Lock and Berkeley transformed Newton's philosophy of science into an empiricism according to which each meaningful non-analytic statement is logically equivalent to some statement involving only terms that refer to sense-experience. This is the verificationist theory of empirical meaning. Every idea about the world must originate either in ideas of sense-experience or be compounded of such ideas. In the hands of Berkeley and Hume it soon became clear that strict adherence to this form of empiricism requires that notions of "matter" and "cause" be grounded in relations of sense experience.

Attempts to carry out in some measure an empiricist meaning analysis were greatly facilitated by the development of modern quantificational logic. But the new logic changed the emphasis of the project from terms to sentences. The project evolved into the plan of setting out the evidentiary relations between statements in which physical object terms are couched and statements of sense experience (or observation). Carnap's *Der logische Aufbau der Welt* (1928) offered verificationist constructions which attempt to show how sentences involving scientific theoretical terms stand in deductive evidentiary relationships to sentences in which only observation terms occur. In this way, Carnap hoped to realize the "elimination of metaphysics" and the "meaninglessness of ontology."[16]

It is natural, therefore, to interpret Russell's philosophy as belonging to the same British empiricism as Newton and Hume. But this interpretation is mistaken. Empiricistic reductive identity may be appropriated in the service of a stance against speculative metaphysics, but there is also a quite different *eliminativistic* approach. Reducitive identifications have a natural ally in empiricism and verificationism. Eliminations, on the other hand, supplant one ontology by another, recovering (at most) the laws of the earlier without identifying the old entities with any of the entities countenanced in the new framework. Eliminations are exemplified by the historical development of physics and chemistry. The eighteenth and nineteenth

[13] See Richard Westfall, *The Construction of Modern Science* (Cambridge University Press, 1977). See also E.A. Burtt, *The Metaphysical Foundations of Modern Science* (Humanities Press, 1952).

[14] See Carl Hempel, *The Philosophy of Natural Science* (Prentice Hall, 1966).

[15] Descartes's substance dualism provided the philosophical foundation of mechanical atomism, though Descartes himself was not strictly speaking an atomist. He thought extension is infinitely divisible.

[16] The derivations were originally to be piecemeal, sentence-by-sentence. Carnap's valiant efforts conclusively corroborated Duhem's thesis that many commonplace theoretical terms (such as "temperature") can only be given conditional (partial) definitions in an observation language.
The unit of empirical meaning is not the term or the sentence, but the entire theory. See Carl Hempel, "The Empiricist Criterion of Meaning," in *Revue Internationale de Philosophie* 4 (1950).

century offered offered a number of theories about subtle aetheric fluids that were highly successful at explaining a wide variety of phenomena. In the process of theory change, the research programs that gave rise to such theories were supplanted by physical theories couched within a new research program that abandons these aetherial fluids. Empirical and conceptual problems pertaining to the aethers (such as its elasticity) became mute, and an entirely new research program, with a new language, and a new set of empirical and conceptual techniques was inaugurated. Many successes of the earlier aether theories were retained by the theories of the new research program. Retention, however, is always partial. The confirmed predictions of an earlier theory in a rival research tradition do not always survive into the supplanting research tradition. Indeed, theoretical processes and mechanisms of earlier theories are at times treated as flotsam.[17] The supplanting tradition may come to regard the terms of the earlier theories as non-referential, or regard earlier ontologies as idle wheels that serve no explanatory purpose.

The supreme maxim of Russell's philosophy of logical atomism is a form of eliminativism, not a form of empiricist reduction. In his paper "On the Relation of Sense-Data to Physics" Russell described his method thusly:

> Given a set of propositions nominally dealing with the supposed inferred entities, we observe the properties which are required of the supposed entities in order to make these propositions true. By dint of a little logical ingenuity, we then construct some logical function of less hypothetical entities which has the requisite properties. This constructed function we substitute for the supposed inferred entities, and thereby obtain a new and less doubtful interpretation of the body of propositions in question. [18]

This is not intended as an epistemological doctrine. Russell's point is that by logical reconstruction, one dissolves the philosophical pseudo-problems generated by the speculative ontologies of the former theory. His scientific method in philosophy offers a re-conceptualization and *logical* reconstruction (where appropriate) of the laws of the earlier theory.

Russell's "supreme maxim" for philosophy, his program for a new scientific philosophy based on logical form, evolved from his work in *logic*. This is clear from the many examples Russell gave of this new scientific philosophy and from his own account of its origins in the philosophy of mathematics.[19] He heralds Frege's analysis of the notion of cardinal number as "the first complete example" of "the logical-analytic method in philosophy."[20] He includes the constructions of his *Principia* (in collaboration with Whitehead) on, propositions, attributes, classes, integers, rational numbers and real numbers. The constructions show that there are no comprehension axioms for propositions, attributes, or classes. There are no natural numbers, no (positive and negative) integers, no rational numbers, no real numbers. The laws of such fields are recovered without assuming an ontology of such entities. Speculative metaphysicians are engaged in a reverie that there are such entities, and in this way *de re* essences and non-logical necessities and confusions are brought into mathematics.

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[17] Larry Laudan, *Progress and its Problems*, (Berkeley: University of California Press, 1977).
[18] *Ibid.,* p. 116.
[19] See Bertrand Russell, "Logical Atomism," in ed., Robert Marsh, *Logic and Knowledge Essays by Bertrand Russell 1901-1950* (London: Allen & Unwin, 1977), p. 326.
[20] Russell Bertrand, *Our Knowledge of the External World* (London: Allen & Unwin, 1969), p. 7.

Russell greatly admires Cantor on infinity and continuity and Weierstrass on the notion of the "limit" of a function.[21] Their studies eventuated in new logical analyses of these notions. With Cantor, the notion of continuity which seemed impossible to render by any notion of magnitude, depends only on the notion of *order*. The *derivative* and the *integral* became, through the new definitions of "number" and "limit," not *quantitative* but *ordinal* concepts. Continuity lies in the fact that some sets of discrete units form a dense compact set. "Quantity," wrote Russell, "… has lost the mathematical importance which it used to possess, owing to the fact that most theorems concerning it can be generalized so as to become theorems concerning order."[22] Weierstrass had banished appeals to infinitesimals in the calculus. He showed that the notion of the "limit" of a function which used to be understood in terms of quantity, as a number to which other numbers in a series generated by the function approximate as nearly as one pleases, should be replaced by a quite different *ordinal* notion. In Russell's view, Cantor's work on the transfinite put to rest centuries of speculative metaphysics surrounding the "infinite" and the notion "continuity." Russell writes: "Continuity had been, until he [Cantor] defined it, a vague word, convenient for philosophers like Hegel, who wished to introduce metaphysical muddles into mathematics. …By this means a great deal of mysticism, such as that of Bergson, was rendered inadequate."[23]

In *A History of Western Philosophy* (1945), Russell even includes Einstein on space-time and the new theory of quantum mechanics as examples of his new analytic method in philosophy. "Physics," Russell tells us, "as well as mathematics, has supplied material for the philosophy of philosophical analysis… What is important to the philosopher in the theory of relativity is the substitution of space-time for space and time." Turning to quantum theory, Russell writes: "I suspect that it will demand even more radical departures from the traditional doctrine of space and time than those demanded by the theory of relativity."[24]

In *Our Knowledge* , Russell endeavors to solve philosophical problems of matter, space and time, by maintaining that there is no "matter" in the traditional sense of rigid bodies, substances, etc. The physical laws governing matter are to be preserved in the constructions without the ontology. Material continuants are replaced by series of *physical* events (called "sense-data"), and this construction incorporates Russell's "perdurance" (or four-dimensional) theory of time and change. Russell is not providing an empiricist and methodologically solipsistic reduction of physics to the data of sense. He is challenging the traditional materialist ontology of physical substances enduring and yet changing through time, re-conceptualizing the notion of 'cause,' solving Zeno's paradoxes, and so on. In 1918 Russell adopted neutral monism which reconstructs both the physics of material continuants (matter persisting through time) and the psychological laws governing unified and persisting minds out of "neutral" physical events of space-time. This scientific method in philosophy (the philosophy of logical atomism) is most salient in Russell's books *Outline of Philosophy* (1927), *The Analysis of Mind* (1921) and *The Analysis of Matter* (1925) . The laws of physics and the laws of psychology (largely the laws acceptable to the then new science of behaviorism) are to be reconstructed without the old ontology of mind and mater.

For Russell, the only necessity is logical necessity. Logical necessity is a matter of structure. Metaphysical conundrums arise because ordinary (and also some scientific) notions

[21] Bertrand Russell, "Mathematics and the Metaphysicians," (printed with the title "Recent Work in the Philosophy of Mathematics," *The International Monthly* 1901. Reprinted in *Mysticism and Logic and Other Essays* (Barnes & Noble, 1976), p. 59-74.
[22] *Ibid*.
[23] Bertrand Russell, *A History of Western Philosophy* (Simon & Schuster, 1946), p. 829.
[24] *Op. Cit*., p. 833.

such as "space," "time," "matter," "motion," "limit," "continuity," "change," and the like, are hybrid notions whose logical and semantic components have not been properly separated from their empirical/physical components. Failure to separate them leads to speculative ontologies, non-logical necessities and essences inhering in special kinds of objects. Russell's analytic philosophy aims at a separation of these components, accomplished by means of a *logical* analysis running side-by-side with advancements and empirical discoveries in physical science. In the process, a new more exacting account of the world emerges which supplants the earlier. On his new scientific method the ontology of an old metaphysical theory is abandoned (or obviated). The constructive replacement retains (where possible) the *structures* given by the laws of the old ontological framework. According to Russell's logical atomism, philosophy provides a conceptual clarification of the concepts of the mathematical and empirical sciences. All necessity is logical necessity. The role of the analytic philosopher is to reveal this.

*2. What not to say about what is not*

To get a sense of the nature of the impetus toward speculative ontology that ordinary language presents to metaphysicians, let us begin with a very old question, dating at least to Plato's *Theaetetus*. The problem concerns thinking about what is not. Plato writes[25]:

> **Socrates:** Then he who thinks of that which is not, thinks of nothing?
>
> Theaetetus: Clearly.
>
> **Socrates:** And he who thinks of nothing, does not think at all?
>
> Theaetetus: Obviously.
>
> **Socrates:** Then no one can think that which is not, either as a self-existent substance or as a predicate of something else?
>
> **Theaetetus:** Clearly not.

Alexius Meinong (1853-1920), an Austrian psychologist working at the University of Graz, became infamous for maintaining that "there are objects of which it is true to say they are not." In 1894 he founded an institute of experimental psychology and supervised the promotion of Christian von Ehrenfels (the founder of Gestalt psychology). Meinong held a strident form of the of Franz Brentano's Principle of Intentionality. The distinctive mark of the mental, Brentano maintained, is that thoughts represent; they are directed toward (are about) objects other than themselves. According to Meinong, an object of thought must have (in some sense) the properties thought about it, else the thought wouldn't be directed to *that* particular object. If one thinks about a winged horse, then there is something (albeit intentionally inexistent) about which the thought is directed - something winged and a horse. A thought about a golden mountain would have its inexistent object be golden and a mountain.

Russell was sympathetic to Meinong's concern about how thoughts are directed, but he found Meinong's form of intentionality "apt to infringe the law of contradiction." A thought about the existent golden mountain, would have as its intentionally inexistent object some-

[25] Plato: *The Collected Papers*, ed., Edith Hamilton and Huntington Cairns (Pantheon Books, 1961), p. 894.

thing, existent, golden and a mountain. A thought about the round square would have its inexistent object be both round and square (non-round).

Perhaps the greatest challenge to any thinker is stating the problem in a way that will allow a solution.[26] One of Russell's achievements in philosophy is to show that the question "How do we think about what is not?" is a complex question. A question is "complex" when the presuppositions that are necessary for its intelligibility are false or do not apply to the case at hand. Suppose a child who is not afraid of the dark is asked: "How did you overcome your fear of the dark?" There is no way to answer. To answer is to attempt to maneuver *within* the parameters set by the presumptions of the question. It is the applicability of the presumptions that must be challenged. But in philosophy, background presumptions are often hidden and difficult to challenge. One of the presumptions of the question as to how we think about what is not, is that we *do* think about what is not. In Russell's view, we do not think about what is not. We think by using quantifiers such as *all* and *some*.

Russell's theory of definite descriptions provided a general means of finding truth conditions for natural language sentences involving an ordinary proper name or definite description without presuming that the name or description has a reference. It was set forth in 1905 in a paper called "On Denoting" and it came to be the centerpiece of Russell's new philosophy of logical analysis.

Russell's theory of definite descriptions can be presented in terms of the following steps toward finding truth-conditions. The steps are these:

Step 1: Replace the given ordinary proper name with an ordinary definite description "the A".

Step 2: Determine the intended scope of the definite description(s) involved.

Step 3: Represent the truth-conditions for the scope quantificationally.

Russell is offering new extension which enables a transcription of "the A is B" as

$$(\exists x)(\mathrm{A}y \equiv_{\mathrm{y}} y = x \;.\&.\; \mathrm{B}x).$$

This is read: "Something is such that it is uniquely A and it is a B." For convenience we can invent a quantifier expression for this, abbreviating as follows:

$$[\iota x \mathrm{A}x][\mathrm{B}x] =_{\mathrm{df}} (\exists x)(\mathrm{A}y \equiv_{\mathrm{y}} y = x \;.\&.\; \mathrm{B}x).$$

Thus, in addition to the quantifiers

*all* $(\forall x)(\ldots x\ldots)$ and *some* $(\exists x)(\ldots x\ldots)$ we have $[\iota x \mathrm{A}x][\ldots x\ldots]$

In this way, the language of logic remains "pure." Its only terms are variables. Transcription from ordinary language into formal logic never employs names or descriptions.

The great benefit of Russell's theory of definite descriptions is its capacity to render scope syntactically (by means of the apparatus of the bound variables of quantification). This is a product of giving quantificational representations of the truth-conditions of ordinary sentences

[26] The source of this phrase is unclear but it has been attributed to Russell.

involving proper names or definite descriptions. Russell offers an entertaining example by taking up the question as to whether the present king of France is, or is not, bald. We have two scopes:

(Scope 1) Something is uniquely king of France at present and not bald.
[ιx*P*x][ ~B*x*]
(Scope 2) It is not the case that something is uniquely king of France at present and bald.
~ [ιx*P*x][ B*x*]

The question is intractable if one accepts that we must examine an intentionally non-existent present king of France - an object of thought which is supposedly referred to by "the present king of France." How are we to choose which, among the myriad possible[27] present non-existent kings of France (some of which are bald and others of which are not bald), is the proper referent of "the present king of France"? Poking fun at Hegel's notion that history progresses by synthesizing contradictories, Russell retorts that, "Hegelians who love a synthesis will probably conclude that he wears a wig."[28]

Consider the question as to whether Pegasus is a horse. In finding the truth-conditions Russell asks us to replace the ordinary name "Pegasus" with an ordinary definite description appropriate to the origins of the name "Pegasus." Let us, therefore, use

The winged horse who according to Greek myth
was born from the beheaded Medusa.

This yields the new sentence

The winged horse who according to Greek myth
was born from the beheaded Medusa is a horse.

Next we give the truth-conditions quantificationally:
Some unique winged horse who according to Greek myth
was born from the beheaded Medusa is a horse.

This is false, since of course, there is no such horse. Naturally enough, however, conversational use of the proper name "Pegasus" may at times demands a more charitable interpretation. One may intend to say the following:

According to Greek myth,
the winged horse who was born from the beheaded Medusa is a horse.

Giving the truth conditions quantificationally yields a somewhat obvious truth:
According to Greek myth,
some unique winged horse born from the beheaded Medusa is a horse.

The different readings can be understood as differences in the "scope" of the quantifier "some." When the truth conditions begin with "some unique…" the scope is said to be *prima-*

[27] In "On What There Is," Quine raises essentially the same question with his example of "the fat man in the doorway."
[28] See *OD*, p. 116. Quine, wondering about non-existent possible fat men in his doorway, alluded to this problem in his 1948 paper "On what There Is."

*ry*, otherwise it is called "secondary." In some cases only a primary scope is available. Consider, for example, whether Pegasus is self-identical. Russell's method renders the truth conditions of "Pegasus is self-identical" as follows:

> Some unique winged horse who according to Greek myth
> was born from the beheaded Medusa is identical to itself.

This is false since there is no such horse. There is no secondary scope available. Contrast "Pegasus is not self-identical". We now have an ambiguity between *primary* and *secondary*. A secondary scope yields:

> It is not the case that some unique winged horse who according to Greek myth was born from the beheaded Medusa is identical to itself.
>
> This is true. The primary scope yields:
> Some unique winged horse who according to Greek myth was born from the beheaded Medusa is such that it is not self identical.
>
> This of course is false since there is no such horse.

Philosophers contrive ontological commitments everywhere. Many derive from the problems of intentionality and thus engage mysteries of mind and thought. Russell's concern is to disabuse philosophers of their fanciful inclinations toward the non-existent. The the scope of definite descriptions is a first step. But it is not the entire story.[29] Consider the following:

> I did not buy the object I hoped to give to you for Christmas
> because it doesn't exist.

At first blush, it may seem that we are pushed into holding that some object of thought—the object I thought I'd give you for Christmas-- is such that it doesn't exist. But an escape is available. We can put:

> Some property is such that I hoped to give to you for Christmas an object that has that property, but I didn't because everything fails to have that property.
>
> Engaging examples of such "objects of thought" abound. Consider this:
>
> Sherlock Holmes is more famous than any existing detective.

Replacing "Sherlock Holmes" by the definite description "the master detective living at 221 Baker Street according to the Conan Doyle stories," we might try the following:

> More people have employed the property of *being the master detective living at 221 Baker Street according to the Conan Doyle stories* in directing their thoughts than they have employed any property that some detective uniquely has.

[29] See Graham Priest, "How the Particular Quantifier Became Existentially Loaded Behind our Backs," in *The Soochow Journal of Philosophical Studies*: Special Issue, ed., Chienkuo Mi (Soochow University, 2007), p. 199.

Finding the right paraphrase is often difficult for want of an adequate philosophy of mind. But Russell has shown us how to preserve a robust sense of reality.

### *3. Russell's Logical Development*

Russell's ontological development is a function of his logical development. His views on ontology are directly tied to his views on what is the proper calculus for logic. As his ideas about logic changed, so also did his ontological commitments. At least one thesis, however, is invariant. It lasts from *The Principles of Mathematics* (1903) through *Principia Mathematica* (1910-1913) and indeed throughout Russell's philosophical thought. It the Russellian ancestor of Quine's maxim "To be is to be the value of a variable." It is this:

> A formal calculus for the science of logic must adopt only one style of genuine variables entity/individual/object variables.

Logic is an abstract science of structure. It is a universal science whose laws apply to all objects alike, be they universals, functions, particulars, abstract, or concrete. The calculus for logic cannot embrace different styles of *genuine* variables. Russell's applications of logic to solve (or better dissolve) philosophical problems, endeavor to supplant one language and ontology by another, recovering the structure of the former without the ontological assumptions that generate philosophical paradoxes. Russell described his technique as "building structure into variables." That is, variables for classes, functions, and the like are not genuine variables but syntactically structured so that the correct logical form is revealed. Ordinary language, of course, contains simple predicates for many concepts, and their grammar makes them appear as if they stand for (simple) properties like any other. Russell's program challenged this. Once the proper truth-conditions are revealed, the conundrums, paradoxes, and the speculative metaphysical doctrines surrounding these notions fall away.

Let us now track the major historical changes in Russell's logical theories and how they impact his ontological development. We shall divide the history into phases.

**Phase I:** From 1903 until 1908, Russell regarded logic to be the general science of propositional structure. Propositions are understood as mind and language-independent states of affairs. Some propositions obtain (are true) and others do not obtain (are false). Every well formed formula in the formal language of logic can be transformed (nominalized) to make a term for a proposition. Propositions are natural candidates as purely logical objects—candidates which can be used to build structured variables which emulate a logical theory of types of attributes and accordingly classes as their extensions.

In *Principles*, Russell offers a bridge between the Aristotelian categorical logic and the use of variables in the quantification theory he learned from Peano at a conference in Paris in 1900. The bridge was based on an ontology of denoting concepts. In categorical logic we have the form "All S are P" and, given Russell's theory of propositions, there is a proposition

'All S are P'.

The constituents of this proposition include the denoting concept 'All S' and the property P. In contrast, there is the quantificational statement "Everything is such that if it is an S then it is a P." This can be represented with the variables of the new quantification theory as

"Every *x* is such that if *x* is an S then *x* is a P." Russell was concerned to offer a theory of the constituents of the corresponding proposition. But this raises the vexing question as to what in the proposition corresponds to the use of the variable "*x*" in the sentence. Russell's theory of denoting concepts attempted to provide a solution. The constituents include the denoting concept 'anything,' and the denoting concept 'every proposition'. They also include the relation of 'substitution,' the property of 'obtaining' and the proposition 'if *a* is an S then *a* is a P.' The proposition is this:

'Every proposition resulting from substituting anything for *a* in

'if *a* is an S then *a* is a P' obtains.'

This leaves the question as to the constituents of the proposition

'If *a* is an S then *a* is a P.'

Its constituents are the properties S and P, the entity *a*, and the relation of 'implication.'

The logical relation of 'implication' is represented by the sign "$\supset$." Where $\alpha$ and $\beta$ are any terms of the formal language for logic, $\alpha \supset \beta$ is a formula. This allows "$x \supset y$" to be a well-formed formula where "*x*" and "*y*" are variables. Complex formulas can be transformed into terms for propositions as well. Thus, for example, the formula "*x* implies *y*" can be transformed (nominalized) into the name "*x* implying *x*" for a proposition. This transformation can be marked by using braces and writing "$\{x \supset y\}$." Thus, for example, Russell has the formula

*x*'s implying *x* implies *x*'s implying *x*.

In symbols this is,

"$\{x \supset y\} \supset \{x \supset y\}$."

Russell also permits general propositions (those named by nominalizing formulas containing bound variables of quantification). Thus the formula

"$(\forall x)$(*x* implies *x*)"

which says that everything implies itself, can be transformed into the term "$(\forall x)$(*x*'s implying *x*)," or using $\supset$ and brackets "$\{(\forall x)(x \supset x)\}$." This feature of Russell's formal theory of logic enables him to define the negation sign thus:

$\sim\alpha$ =df $(\forall x)(\alpha \supset x)\}$.

In this way, Russell characterizes logic as the general science of (propositional) structure. Russell's idea of building structure into variables began in *Principles*. Only individual variables are genuine. And this accords with the view that whatever is, is an individual/entity. Special styles of variables for entities such as propositions can be introduced by explicit defi-

nitions. Thus for example, since according to *Principles* all and only propositions imply themselves, Russell can put:

Prop($x$) =df $x \supset x$
$(\forall p)$A$p$ =df $(\forall x)$(Prop($x$) $\supset$ A$x$).

Russell's official view is that predicate variables (for universals) are also introduced by definitions. This is made possible by his thesis that universals (properties and relations) have both an individual and a predicable nature. The universal *humanity*, for example, occurs in both of the following propositions:

'Socrates is human'
'Humanity belongs to Socrates.'

In the first, it occurs "as concept." In the second, it occurs "as an individual/entity/term." Thus, to quantify over a property Russell relies on the logical equivalence between a proposition of the form 'F$a$' and a proposition of the form 'F belongs to *a'* (or better '*a* exemplifies F' where E 'exemplification' is a primitive relation. Russell can then put:

$(\exists$F$)$(F$y$) =df $(\exists z)$( Property($z$) & E($x$, $z$)).

When it comes to quantifying over two-term relations, Russell demurs, but thinks he needs a more complicated exemplification relation of the form E($x$, $y$, $z$). Thus he has:

$(\exists$R$)$($x$R$y$) =df $(\exists z)$( Relation($x$) & E($x$,$y$, $z$) ).

Russell is aware of Bradley's concern over the problem of the unity of a proposition. But he is protected by the thesis that properties and relations have an indefinable two-fold nature. For instance, the proposition 'F$x$' is logically equivalent to the *distinct* proposition 'E($x$, F).' The latter is not an analysis of the first. Similarly, the proposition '$x$R$y$' is logically equivalent to the distinct proposition 'E($x$, $y$, R).' The latter is not an analysis of the former. The unity (and thereby existence) of a proposition is due to a universal occurring 'as concept." But the universal (property or relation) occurring "as concept" in a proposition is not sufficient to make the proposition obtain. Obtaining (truth) and non-obtaining are primitive unanalyzable properties of propositions.

In *Principles*, denoting concepts played a central role in Russell's solution of the puzzle of how we talk about what is not. Russell had accepted the following schema:

"A is not" is either meaningless or false,

where "A" is any genuine proper name. Because "Pegasus" is regarded as a genuine proper name, Russell reluctantly admitted a distinction between 'being' and 'existence.' Pegasus has being. To say "Pegasus is not (has no being)" either meaningless or false. But we can say that Pegasus does not exist. Thus in *Principles*, Russell accepted that the language of logic has an existence predicate. Interestingly, however, Russell did not think that definite descriptions are

genuine proper names. In *Principles*, he explicitly denies the being of the null-class.[30] There is a denoting concept 'The null class' which occurs in the proposition

'The null class is not (has no being)'.

If, as St. Anselm would have it, "God" is shorthand for "the being a greater than which cannot be conceived," then Russell would not be required (by the theory of *Principles*) to accept the being of God. If, on the other hand, "God" is a proper name, Russell would have to accept God's being - though not His existence. It wasn't long after *Principles*, however, that Russell came to see that many ordinary proper names are not genuinely names but abbreviated descriptions. "Apollo," and all manner of Homeric god names, are not genuine. Russell decided that the only genuine names will occur when a contingent circumstance arises in which a person is immediately acquainted with something - for in such a case existence is assured by the acquaintance. With the theory of denoting concepts buttressed by his new view on proper names, Russell seems close to banishing his thesis that logic must countenance a property of existence.[31]

Russell was never content with the theory of denoting concepts in *Principles* and shortly after its publication in 1904 he came to accept that he had failed to properly analyze the constituents of propositions named by sentences employing the variables of the new quantification theory. By 1905 Russell abandoned his ontology of denoting concepts altogether. But his infamous Gray's Elegy argument against denoting, presented in "On Denoting," defies attempts at a complete understanding to the present day.

As we saw, Russell hoped that denoting concept could form a bridge from categorical logic to the new the variables of the quantification theory. It also played a role in Russell's theory of how we can talk about what is not. It plays this role because denoting concepts, as with universals generally, have a two-fold capacity for occurring in propositions. Consider, for instance, the following propositions:

'All men are mortal'
'*All men* is a denoting concept'.

In the first, the denoting concept 'all men' occurs "as concept." In the second, it occurs "as individual/term/entity" in the proposition The difference in occurrence is displayed by italics but also in the use of the singular "is" in the second expression. But the singular is not always used. Consider,

'Every man is mortal'
'*Every man* is a denoting concept

The difference in occurrence is still present, but suppressed in the ordinary expressions unless we use italics or some other device to indicate the difference. For Russell, the difference is a difference in the structure of the propositions (states of affairs) in question. Indeed, because 'Every man' occurs "as concept" in the first proposition, it is a violation of the propositional structure to imagine removing 'Every man' and replacing it with Socrates (the person).

[30] This seems to have been first made by Cocchiarella. See Nino Cocchiarella, "Meinong Reconstructed versus Early Russell Reconstructed ," *Journal of Philosophical Logic* 11 (1982), pp. 183-214.
[31] See Bertrand Russell, "The Existential Import of Propositions," in ed., Douglas Lackey, *Essays in Analysis by Bertrand Russell* (London: Allen & Unwin, 1973), pp. 98-102.

The linguistic or syntactic replacement of the phrase "Every man" by the expression "Socrates" is perfectly well formed. The linguistic matter of replacing expressions in sentences does not track the ontological issue of the structure of the proposition. The crux of the Gray's Elegy argument of "On Denoting" is that Russell has no viable theory of the difference in the structures of the two propositions.[32]

**Phase II**: Abandoning the theory of denoting concepts left Russell with no account whatsoever of the constituents of the propositions named by nominalizing expressions of the new quantification theory. But Russell's attempt at analyzing such propositions in virtue of substitution was not wholly in vain. It led him directly to both his new theory of definite descriptions and his 1905-1908 substitutional approach to solving the paradoxes of classes and attributes. The substitutional theory was to have been the centerpiece of a second volume of *Principles* which solved the paradoxes. As in *Principles*, bound predicate variables are not genuine, but must be emulated by means of substitutions. What is emulated, however, is a system of simple-type regimented predicate variables.

To understand this let us consider Russell's paradox of attributes and his analogous paradox of classes. The naïve theory of attributes assumed that the following comprehension principle for attributes is logically true:

For every open formula there is an attribute such that an entity has the attribute if and only if it satisfies the formula.

This yields the attribute R which an attribute F exemplifies if and only if it does not exemplify itself. For example, the property of being wise is not wise. Hence, it exemplifies R. The attribute of being self-identical, however, is self-identical. Hence it does not exemplify R.

But what about the attribute R itself? We have the contradiction that R exemplifies itself if and only if R does not exemplify itself. Russell's paradox of classes is analogous. Naïvely one might think the following comprehension principle is a logical truth:

For every condition there is a class of all and only those entities that satisfy the condition.

Now consider the class *r* of all and only those classes that are not member of themselves. The class of all and only wise men is not a wise man and so not a member of *r*. The class of self-identical things is, however, self-identical. Hence it is not a member of *r*. But what about the class *r* itself? The class *r* is a member of itself if and only if it is not a member of itself.

The paradoxes are blocked if logical grammar demands a simple type regimentation. Let *o* designate the lowest type. Simple type theory offers a regimented (or many sorted ) language where variables come with type indices that are sealed off from one another. Statements of predication must be of the form

$$\varphi^{(t1,\ldots,tn)}(\alpha_1^{t1},\ldots,\alpha_n^{tn}).$$

Thus statement that $\alpha$ has the property $\varphi$ must be of the form

$$\varphi^{(t)}(\alpha^{t}).$$

[32] See Gregory Landini, "'On Denoting' Against Denoting," *Russell* 18 (1998), pp. 43-80.

Russell's paradox of attributes cannot be generated since it would require a statement of the form $\varphi(\varphi)$ which violates the simple-type regimentation. The case is analogous for a type-regimented language of classes. In simple type theory, a statement of class membership must have the form

$$A^t \in B^{(t)}$$

Thus the paradox of classes is blocked. It required $r \in r$. which violates simple type regimentation.

A straightforward interpretation of such grammatical restrictions on predicate variables (or class variables) would embrace a simple type hierarchy of attributes (or classes). For instance, on such a view, it is ungrammatical to say that the property of being abstract is abstract. We can only say that the property of being abstract is of type *t* and it has the quite different property of being abstract of type (*t*). But such an interpretation is out of sorts with Russell's that only individual variables are genuine. Russell's task then is to emulate the syntax of simple type theory in a theory that is type free and admits only individual variables as genuine. He achieved this with his 1905 substitutional theory of propositional structure.

Russell's substitutional theory consists of two discoveries. The first, is a way to recover extensional contexts from intensional contexts. This was a by-product of the scope distinctions made possible by the theory of definite descriptions. The second, is a way to use multiple individual variables to emulate a simple type regimented language of predicate variables. Russell begins with a four place relation which he writes as

$$\frac{u}{v}; {}^{x}!\, y$$

This is about entities, and says that *y* results from substituting *x* for *v* in *u*. More exactly, it says that *y* is structurally the same as *u* except containing *x* at each occurrence of *v* in *u*. This enables the formation of definite descriptions for propositions resulting from substituting entities in propositions. The benefit is that Russell can emulate the simple-type regimented expression

$$(\forall \varphi^{(o)})(\forall x^{o})(\varphi^{(o)}(x^{o}) \supset \varphi^{(o)}(x^{o}))$$

by quantifying over two variables *u* and *v* thus:,

$$(\forall u)(\forall v)(\forall x)([(\iota y)(\frac{u}{v}; {}^{x}!\, y)][y \supset y]).$$

In the next type we have:

$$(\forall \varphi^{((o))})(\forall x^{(o)})(\varphi^{((o))}(x^{(o)}) \supset \varphi^{((o))}(x^{(o)})).$$

This is emulated by quantifying with three variables *a, b, c* thus:

$$(\forall a)(\forall b)(\forall c)\,(\forall u)(\forall v)([(\iota y)(\frac{a}{b,c}; {}^{u,v}!\, y)][y \supset y]).$$

The expression

$$\frac{a}{b,c}; {}^{u,v}!\, y$$

is carefully defined as a series of substitutions so that it says that *y* is exactly like *a* except having *u* is all positions of *b* and *v* in all positions of *c*. By using multiple individual variables, simple-types of attributes are emulated within a *type-free* theory whose ontology consists of propositions and universals. On the theory, universals are type free and have both a predicable and an individual nature.

Russell's substitutional theory is a "no-comprehension principles for attributes" theory. It emulates a simple type regimented comprehension principles for attributes. By means of the recovery of extensional contexts in terms of primary scope, it is a "no-classes" theory which emulates a simple-type theory of classes. It offers a genuine *solution* of the paradoxes plaguing the logical notion of an attribute (and class). Its formal calculus for logic is a genuinely universal calculus in the sense that its language is type-free and adopts only one style of *genuine* variables (the entity/individual/object variables). Its laws hold of everything, be it a universal, a particular, or whatever.

Russell's substitutional theory was a monumental achievement and reveals unquestionably that Russell embraced an eliminativistic approach to the philosophy of mathematics. The substitutional language and ontology supplants the language and ontology of a type-stratified theory of attributes in intension (and a type-stratified theory of classes and relations-in-extension).The type distinctions that dismantle the paradoxes are built into the formal re-conceptualization of logical first principles.[33] Classes are not identified with any entities countenanced in the substitutional theory. The theory offers a logical analysis and reconstruction that is ontologically eliminative and structurally retentive. In this way, the major successes obtained by appeal to the existence of classes, the positive constructions of Cantor, Dedekind, Weierstrass, and Frege are retained within substitution. Russell explained that "… the principles of mathematics may be stated in conformity with the theory," and the theory "…avoids all known contradictions, while at the same time preserves nearly the whole of Cantor's work on the infinite"[34] The substitutional theory involves, as Russell put it, "an elaborate restatement of logical principles." The results obtained by appeal to the existence of classes are conceptualized in an entirely new way within the research program of the substitutional theory.

The substitutional theory also reveals that the long standing orthodox opinion that Russell never distinguished syntactic and semantic paradoxes is little more than myth.[35] In 1906, Russell's solution of the syntactic paradoxes (the paradoxes of classes and attributes) was to build type structure into the formal grammar of the theory of propositions. [36] He offered an entirely distinct approach to the semantic paradoxes (Richard, König-Dixon, Berry) of "definability" or "nameability." [37] In fact, he dismisses these as pseudo-paradoxes generated by confused thinking about the notions of "definable" and "nameable." He writes:

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[33] See Appendix B for a brief sketch of how this is accomplished.

[34] "*Les paradoxes de la logique*," *Revue de Métaphysique et de Morale* 14, (1906): 627-50.
The English title is "On 'Insolubilia' and Their Solution By Symbolic Logic," and is reprinted in Douglas Lackey, ed., *Essays in Analysis By Bertrand Russell*, (London: Allen & Unwin, 1973), p. 190-214.

[35] See Gregory Landini, Russell's Separation of Logical and Semantic Paradoxes," in Philippe de Rouilhan, ed., *Russell en héritage*, (Revue Internationale Philosophie 3, 2004), pp. 257-294.

[36] Bertrand Russell, "On 'Insolubila' and Their Solution By Symbolic Logic," p. 209. See also Bertrand Russell, "On the Substitutional Theory of Classes and Relations," in Douglas Lackey, ed., *Essays in Analysis By Bertrand Russell*, (London: Allen & Unwin, 1973), p. 186.

[37] Gregory Landini, "Russell's Separation of the Logical and Semantic Paradoxes," in Philippe de Rouilhan, ed., *Russell en héritage*, (Revue Internationale Philosophie 3, 2004), pp. 257-294.

> A definition, symbolically, consists of a certain finite number of marks. Such marks must either by the symbols of our undefined fundamental terms, or brackets, or letters standing for variables of which all values are to be taken. ...If the number of fundamental notions is *c*, the number of definitions containing *n* marks is therefore certainly not greater than the number of permutations of $2n + c$ things taken *n* at a time. This number is finite; the sum of all such numbers for different finite values of *n* is $\aleph_0$. Now the cardinal number of ordinals of the second class exceeds $\aleph_0$; hence some of these must be indefinable, and among those that are indefinable there must be a least. But this ordinal seems to be defined as 'the immediate successor of the ordinals that are definable'. At first sight this looks like a contradiction, but in fact, it is not. For although every individual number less than this one is definable, the whole class of them is not definable. It *seems* to be defined as 'the class of definable ordinals'; but *definable* is relative to some given set of fundamental notions, and if we call this set of fundamental notions I, 'definable in terms of I' is never itself definable in terms of I.
>
> ...It is easy to define "definable in terms of I" by means of a larger apparatus I*; but then "definable in terms of I* " will require a still larger apparatus I** for its definition, and so on. Or we may take "definable in terms of I" as itself part of our apparatus, so that we shall now have an apparatus J consisting of I together with "definable in terms of I". In terms of this apparatus J, "the least ordinal not definable in terms of I" is definable, but "the least ordinal not definable in terms of J" is not definable. Thus the paradox of the least indefinable ordinal is only apparent.[38]

In Russell's view, the notion "definable" is incoherent. Russell anticipates a Tarski-style hierarchy of notions "definable in I," where I is a language whose signs and formation rules are fixed. This notion of "definable in I" does not generate paradox.

Quine is famous for arguments rejecting type-regimented languages. And Wang has shown that many-sorted languages are translatable into one-sorted languages without loss of expressive power or provability (though the reverse does not hold). We can always express statements formed with multiple sorts of variables in a language with only one sort of variables (individual variables) by adopting appropriate predicates such as "*x* is a thing," and "*x* is a class," and "*x* is a number." Quine's observes that even the language of type-theory succumbs. Quine writes:

> …we can even abandon Russell's notion of an hierarchical universe of entities disposed into logical types; nothing remains of type theory except an ultimate grammatical restriction on the sorts of repetition patterns which variables are allowed to exhibit in formulas. Yet formally our logic, refurbished as described, is indistinguishable from Russell's theory of types plus Russell's convention of typical ambiguity. Now the point of this logical digression is that even under the theory of types the use of distinctive styles of variables, explicitly or even implicitly, is the most causal editorial detail. … It is a distinction which is not invariant under logical irrelevant changes of typography.[39]

Quine proposes that instead of using indexed letters as variables specifically for entities of type t one could adopt a predicate "$T^t(x)$" to mean that "*x* is an entity of type t." Thus instead

[38] Bertrand Russell, "On the Substitutional Theory of Classes and Relatins," in ed., Douglas Lackey, *Essays In Analysis By Bertrand Russell* (Allen & Unwin, 1973), p. 185.
[39] *Op. Cit.*, p. 210.

of $(\forall \varphi^t)\, A\varphi^t$ one has $(\forall x)(T^t(x) \supset Ax)$. Applying similar techniques to the type-theory of classes, Quine writes: "This does away with Russell's grammatical restriction which declared '$x^m \in y^n$' meaningless where $m+1 \neq n$. Sense is now made of '$x^m \in y^n$' for all $m$ and $n$. If $m+1 \neq n$, then '$x^m \in y^n$' merely becomes false. That we can suddenly be so cavalier with Russell's grammatical restriction makes one wonder whether he needed to make it. He did not."[40]

Quine regards the use of many sorted languages as a "trivial consideration," "a casual and eliminable shorthand," and "not invariant under logically irrelevant changes in typography."[41]

Quine rejects what he took to be Russell's type-theory of *entities*. But he misunderstood Russell's views on ontology. Russell did not have a type-theory of entities! Russell's substitutional theory is no "trivial consideration." It endeavors to get the ontology right, and the right ontology is one that does not embrace comprehension principles for attributes (or classes). It is a no-classes theory. Quine's criticisms are more properly directed at Carnap's attempt to achieve the meaninglessness of philosophical ontology by adopting languages with many-sorted variables. In "Empiricism, Semantic and Ontology," Carnap walks in the footsteps of Russell in holding that "...exists" is a pseudo-predicate. Carnap hopes to realize his doctrine of the meaninglessness of questions of philosophical ontology by treating "... is a physical object" in just the way Russell treated "$x$ exists." Both are pseudo-predicates. Ontological assertions thereby vanish into variables themselves. In a similar way, Carnap hoped to eliminate "…is a number" in favor of adopting a language with special style of variables sealed off from the individual variables. Consider the notion of arithmetic evenness. If we introduced special styles of variables, *m, n*, etc., we could define the notion thus:

$$\text{Even}(m) =_{df} (\exists n)(\, n \times \mathbf{2} = m).$$

Quine objected to this. The special style of variables are a "casual editorial detail" eliminable by adopting predicates such as "$x$ is a number" and one style of individual variables. Thus:

$$\text{Even}(x) =_{df} (\exists y)(\, y \text{ is a number } \&\; y \times \mathbf{2} = x).$$

In stark contrast, consider the type-regimented definition:

$$\text{Even}^{(((o)))}(\beta^{((o))}) =_{df} (\exists \alpha^{(((o)))})(\, N^{(((o)))}(\alpha^{((o))}) \;.\&.\; \alpha^{((o))} \text{ X } \mathbf{2} = \beta^{((o))}).$$ [42]

In Russell's substitutional theory, this is emulated as follows:

$$\text{Even}(x, y, z) =_{df} (\exists h, d, e)(\; N(h, d, e) \;.\&.\; \iota(\tfrac{s}{t,w})[\tfrac{s}{t,w}; r, c \;.\approx_{rc}. \tfrac{h}{d,e}; x, y \times_{xy} \mathbf{2}xy)(r, c)] = \iota(\tfrac{s}{t,w})[\tfrac{s}{t,w}; r, c \;.\approx_{rc}. \tfrac{x}{y,z}; r, c\,])$$

Technical details of the substitutional theory aside, arithmetic *evenness* is not a one-placed property. It is a three-placed relation of entities $x$, $y$, and $z$. [43] On this view of the truth-conditions, it is clear that "The moon is arithmetically even" is a pseudo-statement.

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[40] W.V.O. Quine, *Set Theory and Its Logic* (Cambridge: Harvard University Press, 1980), p. 268.
[41] Ibid., p. 208.
[42] We have omitted discussion of Russell's no-classes theory which uses lower-case Greek for the construction of extensional contexts.
[43] See Gregory Landini, Russell's Hidden Substitutional Theory (Oxford: 1998).

Carnap hopes to follow the Russellian approach, but he endeavors to achieve it by simply adopting many-sorted languages with different styles of variables. Russell can argue for his analysis of the truth-conditions by proclaiming that it gets the ontology right. But for Carnap, ontology is meaningless. Like Russell, Carnap regarded "*y* is a number" as pseudo-predicate of a muddled philosophical ontology. But unlike Russell, Carnap is left with little to say when it comes to why he singles out this predicate for elimination as a pseudo predicate. Russell's thesis was the only necessity is logical necessity. Thus necessities have to be analyzed away as either a product of confusion or grounded in the ontology of logic. Advocating the meaninglessness of philosophical ontology, Carnap cannot appeal to the ontology of logic. He was a conventionalist about logical truth. As Quine points out, Carnap must appeal to a notion that the pseudo-predicates are those involved with "analytic" (or "necessity") truths distinct in kind from synthetic (empirical) truths. Couched within conventionalist views about logic and metaphysics, however, Carnap cannot make good on the distinction. Thus, Quine has a telling objection to Carnap's attempt to eliminate ontology in favor of structured variables. But Quine has no similar objection to Russell's eliminativism.

Russell's substitutional theory was to have been the crowning feature of the proposed second volume of *Principles*. It was to have provided a solution of the paradoxes plaguing Russell's logicism (the thesis that mathematical truths are among the logical truths). But the second volume was never written. Phase II of Russell's ontological development reveals why. In April of 1906 Russell discovered a paradox unique to the substitutional theory which I have called the "$p_o/a_o$ paradox." Russell's discovery of this paradox is the most important event in the historical evolution of *Principia.* This paradox eventually caused Russell to abandon his ontology of propositions altogether. Thus 1908 marked the end of Russell's substitutional theory of propositional structure. The philosophy and ontology of *Principia* are the direct result.

Russell's propositions were intensional entities not intentional ones involving semantic notions such as "truth," "reference" or "designation." The $p_o/a_o$ paradox that spoiled the substitutional theory is syntactic in nature. Russell came to believe that to avoid it he would have to introduce a stratified language with variables restricted to orders. A theory of orders of entities, however, would undermine the legitimacy of the theory's claim to be a universal science of logic. It is incompatible with Russell's doctrine of the unrestricted variable and type-freedom.

The "$p_o/a_o$ paradox" is unique to substitutional theory. But it resembles a paradox that Russell had investigated Appendix B of *Principles* which arises when a theory of propositions is paired with a theory of classes, or even a theory according to which attributes have only a predicable nature (and thus predicates cannot occur in subject positions)[44] Simple type theory is powerless to block this paradox.. But in the substitutional theory, there are no comprehension principles for class or attributes. Thus the $p_o/a_o$ paradox was different and unexptected.

Russell tried hard to solve it in several unpublished manuscripts of 1906 and 1907. At one point, Russell thought he had found a solution by abandoning his ontology of general propositions. In the new theory, only quantifier-free formulas can be nominalized to form terms for propositions. This solution occurred in Russell's September 1906 paper "On 'Insolubilia' and Their Solution by Symbolic Logic." Russell succeeds in avoiding the $p_o/a_o$ paradox, but unless special new "mitigating axioms" are introduced the new system is too weak to recover the substitutional emulation of the simple-type theory of attributes from which mathematics is to be developed. After the paper was published, Russell came to see that these mitigating axioms

[44] Gregory Landini, "New Evidence Concerning Russell's *Substitutional Theory* (Oxford, University Press, 1998).

enable a new version of the "$p_o/a_o$ paradox.." In his 1907 paper "Mathematical Logic as Based on the Theory of Types," Russell sketched a different way to avoid the $p_o/a_o$ paradox. Instead of abandoning general propositions, he introduces a hierarchy of orders of propositions.[45] The formal language of substitution now includes many sorts of variables adorned with order subscripts for the hierarchy of orders of propositions. New axioms assure the "reducibility"of the orders of propositions in extensional contexts. Unlike the earlier theory, the new axioms of propositional reducibility form a consistent theory strong enough to emulate a simple-type theory of attributes. More exactly, the new substitutional theory emulates what is now known as Russell's "ramified type-theory of attributes.."

**Phase III:** In 1908 Russell' s ontological development entered a new phase because he was not satisfied with an ontology of orders of propositions. *Principia Mathematica* is not the substitutional theory with orders of propositions. Russell contemplated putting the substitutional theory into an appendix of *Principia*, but this never occurred. *Principia* marks an important shift in Russell's thinking. He broke away from his long held view that logic is the general science of propositional structure. Logic is the general science of structure. But according to *Principia* there are no propositions. In *Principia,* the ontology is more austere than ever.

*Principia* also marks a shift in Russell's endeavor to build structure into variables.[46] The syntactic orientation so manifest in the substitutional theory gives way to a semantic phase that adopts many sorted predicate variables "internally limited by their significance conditions." The elimination of propositions facilitated this shift. The formal language of *Principia* permits predicate variables adorned with order\type indices, but these variables are not genuine. They are semantically interpreted in terms of a recursively defined hierarchy of senses of "truth" and "falsehood." The base of the recursion is Russell's famous "multiple-relation theory of judgment" according to which an atomic belief is true if and only if it corresponds to a *fact.*

It is important to understand that Russellian facts and propositions are like men and dinosaurs. They never coexisted. Obtaining (truth) and non-obtaining (falsehood) are unanalyzable properties of propositions (akin, as he puts it, to the whiteness and redness of roses). One believes truly when one believes a true proposition; one believes falsely when one believes a false proposition. In *Principia*, however, matters are quite different. There are no propositions. One believes truly when there exists a fact corresponding to one's belief state (which is itself a fact). The notion of a fact *obtaining* (being true) is incoherent. Thus, facts cannot be identified with obtaining (true) propositions. Russell is explicit in *Principia*. There are no propositions.

*Principia* is an eliminativistic theory. Russell's motivefor the constructions of *Principia* are no different than his motives for the constructions of the substitutional theory. The motive derives from his conception that any calculus for the science of logic must be type and order free. But Russell intended *Principia* to be more radically eliminative than the substitutional theory. In *Principia*, logic is to proceed without comprehension principles for attributes, classes, or even propositions. *Principia* espouses a logical re-construction which endeavors to philosophically explain and justify order/type indices on predicate variables without appeal to any orders or types of *entities*. The explanation builds the structure of orders

[45] The paper took a long time to be published and appeared in1908.
[46] See Gregory Landini, *Russell's Hidden Substitutional Theory* (Oxford, 1998)

into a nominalistic (substitutional) semantics for predicate variables. [47] In short, the order indices on predicate variables are explained by appeal to a hierarchy of orders of languages. The order index on a predicate variable reflects the truth-conditions for the formulas of the language in question. Type and order indices on the predicate variables of the language of *Principia* are explained away as "limitations built into the conditions of significance" of the use of the predicate variables. Only the individual variables of *Principia* are genuine variables; they are type-free since they range over whatever there is, universals, facts, concrete particulars. *Principia's* predicate variables are *not* genuine.

On the heels of *Principia*, Russell endeavored in 1912 and 1913 embarked on a project of refining the multiple-relation theory of truth. Russell was eager to solve outstanding problems that plagued *Principia's* recursive definition of "truth" as correspondence to a fact. General sentences such as "Everything is such that if it is F then it is G" are made true by many facts—one for each value of the variables. The truth-conditions of general sentences work their way down to the truth-conditions for quantifier free atomic sentences. It is important to note that the recursive account was intended to apply only to the sentences of the formal language of *Principia*. Those sentences never involve ascriptions of propositional attitudes such as belief. A recursive definition certainly would not be appropriate to the truth conditions for sentences ascribing belief. (Obviously, "S believes p or q" cannot be given the truth condition that either S believes p or S believes q".)

The recursive theory of truth was the foundation of *Principia's* semantic explanation of the order component of the order\type indices on the predicate variables of the formal language. It was part of a nominalistic semantics for predicate variables that interprets them as ranging over a fixed set of formulas at a level in a hierarchy of languages. The atomic or base case of the recursion is handled by Russell's multiple-relation theory. The truth-conditions for atomic sentences are given by providing a definite description for the fact that would correspond to a discursive episode of belief (or understanding) that would be held by a person asserting the atomic sentence. The negation of that atomic sentence is made true by the absence of any fact corresponding to that episode of belief (or understanding). For example, Othello's episode of belief with respect to his assertion of the sentence "Desdemona loves Cassio" is *Othello's believing with respect to Desdemona, 'loves', and Cassio*.

The atomic sentence "Desdemona loves Cassio" is true if and only if there is a fact corresponding to this episode of belief—that is, there is a fact which consists of Desdemona, 'loves' and Cassio (in the proper structure). The sentence "Desdemona does not love Cassio" is true when it is not the case that there is a fact corresponding to the episode of belief.

Russell soon recognized that such definite descriptions are problematic because some facts are "permutative." Consider for example, the following two distinct facts:

'*Desdemona's loving Cassio*'
'*Cassio's loving Desdemona*'.

These two facts have precisely the same constituents, namely, Cassio, Desdemona, and the relation '*loves*.' What makes the facts different is the structuring (ordering) of these constituents. What is it, then, that makes Othello's belief episode point to the first, and not the second, as its properly corresponding fact?

In *Problems of Philosophy* (1912*)*, Russell imagined solving the problem by appeal to a partial isomorphism between a given episode of belief and a properly corresponding fact.

[47] The individual variables of *Principia* are treated objectually, however. Universals (properties and relations in intension) have both an individual and a predicable nature and are counted among individuals.

Othello's episode of belief is itself a fact which has a structure imposed by the relation of *'belief'* which occurs in it in a way that relates the other constituents into a complex unity. This is structurally isomorphic with part of the fact '*Desdemona's loving Cassio'*

That is, the ordering imposed by the four-term relation of '*belief*' is similar to that imposed by the two term relation of '*love.*'

In his unfinished book *Theory of Knowledge* (1913), Russell became dissatisfied with this view. Isomorphism can only make sense if in a fact there is a *first* constituent, a *second* constituent, and so on. Perhaps some spatial facts have this feature, but there is no 'first' and 'second' in facts of love. In 1913 Russell realized that the definite description must describe the order of the constituent in the properly corresponding fact. To do this, Russell maintains that the relating relation of a fact induces in it " position relations" that the constituents of the fact have to the fact. There is no "first" position, "second" position, etc., but there are nonetheless positions. Thus the truth conditions for "Desdemona loves Cassio" are given by saying that the sentence is true if and only if there is a fact $\alpha$ and position relations P1 and $P_2$ determined by the relation of '*love'* such that Desdemona has $P_1$ in $\alpha$ and Cassio $P_2$. in $\alpha$. Of course this leaves the truth conditions for "Desdemona has $P_1$ in $\alpha$" But this can be treated by the recursion since it does *not* involve the vexing problem of defining correspondence with permutative facts.

Russell also tentatively introduces *logical forms* for monadic predication, dyadic predication, etc., to account for the believer's *understanding* of the structure of the purportedly corresponding fact. What entities are logical forms? In *Theory of Knowledge* Russell suggests that they be indentified with *fully* general facts that are so abstract that they have no constituents. For example, the logical form of dyadic predication is the fact *Something's standing in some relation to something*.

In his 1917 Logical Atomism lectures at Harvard University, Russell goes further still. He accepts partly general facts and even negative facts. It is important to understand, however, that these general facts, negative facts, and are not truth-makers for general sentences, negative sentences. They play a central role in understanding of logical notions such as "not," "or," "and" and "all," but they do not spoil the recursive definition of "truth" and "falsehood" set out in *Principia*. As Russell famously put the matter, what we understand in using "all" cannot be a conjunction, for we need to add that these are *all* the relevant conjuncts. But this point is not in tension with *Principia's* thesis that many facts conspire to be the truth-makers for general sentences, not general facts.

Logical forms play a central role in understanding logic. They also play a central role, in the truth-conditions for statements ascribing propositional attitudes. Following the plan of *Theory of Knowledge*, the truth conditions for the sentence "S believes that all men are mortal" are given by forming a definite description of a fact (an episode of belief) consisting, in proper order, of S, the relation of *'belief'* Humanity, Mortality, and the logical form of *generality*. In turn, the person S believes truly, when all men are mortal. None of this conflicts with *Principia's* recursive definition of "truth" and "falsehood." We must never think that the truth-condition for a sentence involving quantifiers corresponds to a general fact. Only atomic facts are truth-makers. This is the heart of the recursive theory which is the very foundation of *Principia's* explanation of the order indices on its predicate variables.

During this period of intense work on a recursively defined correspondence theory of truth, Russell embarked on a large epistemological project. He hoped to offer an new epistemological theory which could transcend the empiricism-rationalism debate that so animated Kant. Russell found Kant's transcendental idealism odious and antithetical to logicism. Russell's new epistemological theory was build upon his foundational thesis that a mind can stand in a

relation of '*acquaintance*' to universals, a sense-data (as physical momentary particulars), facts, and logical forms. Russell hoped that his theory of acquaintance could provide the foundation of all knowledge, *a priori, a posterior*, logical and empirical. Acquaintance with universals and logical forms makes it possible to have descriptive knowledge of the matter of the empirical world with which we can never have acquaintance..

**Phase IV:** By 1918 Russell abandoned the multiple-relation theory which was to be the foundation of his correspondence theory of truth. Russell abandoned his theory that a mind can stand in a primitive relation of *acquaintance* to objects such as concrete particulars, universals, sense-data and logical forms. This marks the final phase of Russell's ontological development.

When he converted to neutral monism, Russell decided that universals are distinguishable only by their causal powers and have a predicable nature only. A predicate can only occur in a predicate position and never in a subject position. Thus, Russell had to abandon his thesis that a mind can stand in a relation of 'acquaintance' to a universal. He had to abandon the multiple-relation theory, which assumes that a universal can occur non-predicationally as a constituent of a fact. This made the multiple-relation theory impossible. The multiple-relation theory requires definite descriptions of facts in its efforts to recursively define "truth" as correspondence. Any such definite description would put a predicate in a subject position. Moreover, Russell came to accept that his student Wittgenstein had rightly objected to his growingly weighty ontology which now included, general facts and logical forms. To redesign the correspondence theory of truth, Russell sought a new philosophy of mind in sympathy with some of the then new and exciting ideas of the scientific psychology of behaviorism.

These are radical changes. Indeed, long influenced by McTaggart's treatise on the unreality of enduring objects in time, Russell endeavored to eliminate both the concept of *matter* of traditional materialism and the concept of *mind* of traditionalist mentalistic psychology. The laws of physics and the laws of psychology are to be preserved, where possible, without the ontology of matter and mind. This is a radical change indeed. Russell came to be strongly influenced by Einstein's special relativity (1905), Minkowski's space-time diagrams (1907) and the general theory of relativity (1915). Emboldened by Eddington's interpretations of relativity,[48] Russell thought that Einstein's new physics has very important implications which destroy traditional philosophies of mind, matter, space and time. Einstein's special relativity held that coordination by light propagation is the standard for any possible empirical application of the concepts of *distance* and *time* to physical processes. The result is that any temporal ordering of events is relative to the behavior of light. If a light source is in the center of a box moving, as measured from the roadside, at close to light speed, then when the light is turned on, it will hit the wall of the box opposite the direction of its motion before it hits the wall on the opposite side. But from the frame of reference of the center of the box, the light hits both walls simultaneously. Thus the temporal ordering of events is frame relative. Of course, Lorentz famously tried to resist this conclusion and retain something of the tradition notion of matter, postulating that bodies contract when they undergo accelerations close to light speed. Einstein rejected this. The rigidity of a body, its length and its mass (its propensity to resist change of its motion) are not, according to Einstein's theory, invariant in all reference frames.

Russell held that converting to Einstein's relativity mandates the abandonment of the traditional notion of a *material substance*. There is no matter in the traditional sense of a rigid

[48] Sir Arthur Eddington, *The Nature of the Physical World* (Ann Arbor: University of Michigan Press, 1958).

body with fixed length and mass persisting through time. The laws of the physics of "matter" must be reconstructed without the ontology. On Russell's view, physics begins from the notion of an event—a process. Light co-ordinations between events give physical determinacy to the notion of one event *overlapping* in whole or part another event. If the overlapping is fine-grained enough[49], we can apply the calculus-- the Weierstrass construction of the notion of a *limit*. We can then construct the notions of a *point particle* and a *temporal moment.* Russell rejects McTaggart's A-series according to which an event changes its temporal properties (*future* then *present* then *past*) in favor of the B-series which orders events *at* times.

Thus Russell's neutral monim is really quite unlike that of his famous predecessors, Spinoza and James. Russell's scientific philosophy of logical atomism came to embrace a neutral monism that is form of four-dimensionalist physicalism. The neutral "stuff" are events of space-time, and both the laws of physical continuants "persisting" through temporal change, as well as the psychological laws (largely behavioristic) of unified 'selves', are recovered by logical ingenuity. As early as 1901 Russell offered the following amusing passage:

> Weierstrass, by strictly banishing from mathematics the infinitesimal, has at last shown that we live in an unchanging world, and that the arrow in its flight is truly at rest. Zeno's only error lay in inferring (if he did infer) that, because there is no such thing as a state of change, therefore the world is in the same state at any one time as at any other. This is a consequence which by no means follows; and in this respect, the German mathematician is more constructive than the ingenious Greek.[50]

Now finally in the 1920's the new theory of relativity (and the then fledgeling quantum theory) seemed to him to conclusively corroborate the view. Russell aptly described his ontological development as a "retreat from Pythagoras" or better a retreat from Plato. There are no propositions, no denoting concepts, no-non-existents, no-classes, no-numbers, no-attributes (with both a predicable and individual nature), no matter and no mind. Ultimately, Russell's ontology consisted of universals (as causal powers with only a predicable nature), and ordered series space-time events. By the 192o's he had arrived at positions which anticipate the scientifically oriented naturalization of epistemology and philosophy of mind current today.

[49] It is an unanswered empirical question whether the overlapping of events is, in fact, fine-grained enough to enable the construction of a continuous series. They do seem sufficient to enable the construction of a series order isometric to the rationals. See Bertrand Russell, "On Order in Time,"*Proceedings of the Cambridge Philosophical Society*, vol. 17 (1935), pp. 441-449.

[50] Bertrand Russell, "Mathematics and the Metaphysicians," in Mysticism and Logic (New Jersey: Barnes and Noble, 1976), p. 63.

**Chart of Russell's Ontological Development**

Phase I: 1903

- Universals
- Denoting concepts
- Classes (tentatively)
- Propositions
- No comprehension axioms for attributes
-

Phase II: 1905-1908

- Universals
- Propositions
- No Denoting concepts
- No Classes (tentatively)
- No comprehension axioms for attributes
- No general propositions (1906)
- Hierarchy of orders of general propositions (1908)

Phase III: 1910-1917

- Universals
- Facts
  - Logical Forms (1913)
  - Negative (1917)
  - General (1917)
- Sense-data (1914)
- Minds (subject, though perhaps not the 'self' persisting in time)
- No Propositions
- No Denoting concepts
- No Classes (tentatively)
- No comprehension axioms for attributes (propositional functions)
- No general propositions (1906)
- No Hierarchy of orders of general propositions (1908)

Phase IV: 1918-

- Universals (predicable nature only)
- Facts (events of space-time including "images")
  - Negative
  - General
- No-sense data
- No Logical forms
- No Propositions
- No Denoting concepts
- No Classes (tentatively)
- No comprehension axioms for attributes (propositional functions)
- No general propositions (1906)
- No Hierarchy of orders of general propositions (1908)
- No matter (material continuants persisting in time)
- No minds (self-consciousness unified and persisting in time)

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